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The Condition for Contact Grasp Stability

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Abstract

We distinguish between two types of grasp stability, which we call spatial grasp stability and contact grasp stability. The former is the tendency of the grasped object to return to an equilibrium location in space; the latter is the tendency of the points of contact to return to an equilibrium position on the object's surface. We show via examples that spatial stability cannot capture certain intuitive concepts of grasp stability and hence that any full understanding of grasp stability must include contact stability. We derive a model of how the positions of the points of contact evolve in time on the surface of the grasped object in the absence of any external force or active feedback. From this model, we obtain a condition which determines whether or not a two-fingered grasp is contact stable.

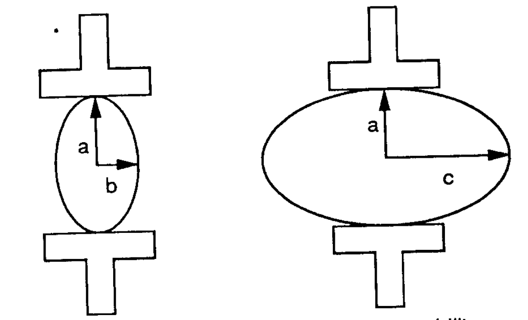
1 Introduction

Stability is the tendency of a system to return to an equilibrium state when displaced from this state. Currently, there are two different views of what constitutes the state of a grasp, leading to two different concepts of grasp stability. The first views the state as the position (and velocity) of the grasped object relative to the palm of the hand. Hence, grasp stability refers to the tendency of the object to return to its original position when displaced by an outside force. We call this *spatial grasp stability*. The second views the state as the position (and velocity) of the points of contact on the surfaces of the object and the fingers. Hence, grasp stability is the tendency of the points of contact to return to their original locations in response to a disturbance. We call this *contact grasp stability*.

Both of these types of grasp stability have been investigated in the past. For spatial grasp stability, Salisbury [15] defined a quantity called the grasp matrix and showed that if this matrix has sufficient rank then the fingers can apply force and torque in arbitrary directions. (Under such conditions the grasp is said to have the property of *force closure*.) The system (whose state is the position and velocity of the object) is hence locally controllable and can be stabilized by an appropriate control law. (Nguyen [13] discusses a class of control laws which stabilize a grasp in this sense.) Salisbury's force-closure criterion only categorizes a grasp as (spatially) stable or unstable and does not distinguish among the huge number of spatially stable grasps. For the purposes of grasp selection, we do need to make such a distinction, and there have been a few numerical criteria proposed for spatial grasp stability, e.g. [6, 7, 8, 14].

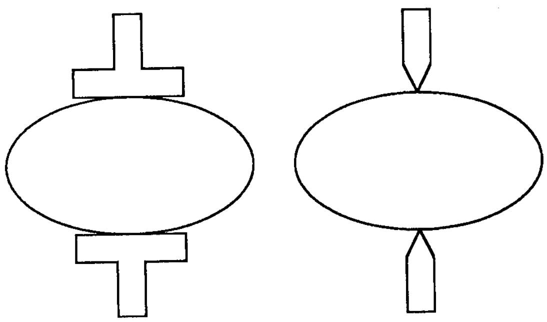
For contact grasp stability, Hanafusa and Asada [5] built a hand whose fingertips slid along the surface of a grasped object in such a way as to minimize a mechanical potential energy. Hence, in response to any small displacement of the contact points, the system would return to its equilibrium position, and the grasp would therefore be (contact) stable. (Nguyen [13] also discusses the stability of sliding contact.) However, such a hand is fundamentally different from human hands in that its fingertips are designed to slide easily across the object surface while human fingertips exert frictional forces. (Friction in fingertips is generally a good thing because it allows spatially stable grasping with far fewer points of contact.) Cutkosky and Wright [3] analyze contact grasp stability for human-like fingertips and show how the viscoelastic nature of human fingertips enhance this stability. However, they do this analysis for only a few special cases.

In this paper, we extend the analysis of Cutksosky and Wright to derive a general condition for contact stability of a two-fingered grasp of an arbitrary three-dimensional object. Through examples, we see how this condition embodies many intuitive notions about grasp stability, concepts that measures of spatial stability cannot capture.



(a) b<a => instability (b) c>a => stability

Figure 1: Object curvature effects grasp stability.



(a) flat fingertips =>stability (b) pointed => instability

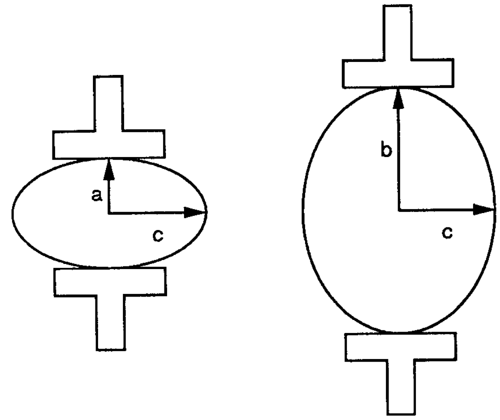
Figure 2: Finger curvature effects grasp stability.

2 Aspects of grasp stability

In this section, we examine examples which illustrate certain important aspects of grasp stability and why measures of spatial stability cannot capture them.

Example 1 Consider two ellipsoids, one given by the equation $x^2/a^2 + y^2/b^2 + z^2/b^2 = 1$ and the other by the equation $x^2/a^2 + y^2/c^2 + z^2/c^2 = 1$, where $b < a < c$. Consider grasping these ellipsoids with flat fingertips at the points $(a, 0, 0)$ and $(-a, 0, 0)$. (Figure 1 shows a cross-section of these grasps.) From the point of view of spatial stability, these two grasps are equivalent because the grasp matrices are identical and the finger Jacobeans are identical. However, the second grasp seems more stable. This example illustrates the importance of the curvature of the grasped object to grasp stability.

Example 2 Consider two different grasps identical in all respects except the shape of the finger surfaces. In one case the fingers are flat, while in the other case the fingers are pointed. (Figure 2 illustrates these two scenarios.) As in the last example, the grasp matrices and finger Jacobeans are identical; therefore, these grasps are equivalent from the point of view of spatial stability. However, the first grasp seems more stable. Hence, the curvature of the fingers effects grasp stability.



(b) c>a => stability (b) c<b => instability

Figure 3: Distance between contacts effects stability.

Example 3 Consider two ellipsoids, one given by the equation $x^2/a^2 + y^2/c^2 + z^2/c^2 = 1$ and the other by the equation $x^2/b^2 + y^2/c^2 + z^2/c^2 = 1$, where $a < c < b$, and consider grasping these objects with flat fingertips. Let the grasp points be $(a, 0, 0)$ and $(-a, 0, 0)$ in the first case and $(b, 0, 0)$ and $(-b, 0, 0)$ in the second case. (Figure 3 shows these two scenarios.) Then, the curvatures of the objects at all the points of contact are identical and equal to $\mathbf{diag}(1/c, 1/c)$. The shapes of the finger surfaces are also the same in the two cases. The first grasp seems more stable than the second despite the objects and the fingers having the same local geometry because the points of contact are closer to each other in the first case than in the second case.

Example 4 Consider two grasps which are identical in all ways except the material properties of the fingers. In the first case, the finger surface is either rigid or purely elastic, while in the second case the finger surface is viscoelastic, or in more common language "soft" or "squishy". The second case seems to provide a more stable grasp [1, 3].

We have thus identified four factors which seem intuitively to be important in grasp stability: (1) object shape, (2) finger shape, (3) distance between the points of contact, and (4) finger and object viscoelasticity. We will below derive a measure of grasp stability which incorporates all of these factors. However, we first review contact kinematics.

3 Contact kinematics

In the nomenclature of [11], *contact kinematics* refers to the evolution of a point of contact on the surfaces of two objects in response to a relative motion of these objects. We now present in a simplified form some concepts and equations from the study of contact kinematics relevant to our analysis of grasp stability.

Consider two rigid objects, obj_1 and obj_2, which have a single point of contact.

Definition 1 A *local reference frame* for this point of contact is a right-handed, orthonormal coordinate frame whose origin is at the point of contact and whose z axis is the outward normal to obj_1 at this point. (The x and y axes can be any unit vectors satisfying the constraints of orthogonality and right-handedness.)

Definition 2 The *curvature* of a surface S at a point $\vec{s} \in S$ relative to two unit vectors $\vec{x}$ and $\vec{y}$, which are orthogonal to each other and tangent to S at $\vec{s}$, is the 2x2 matrix K such that any infinitesimally small displacement $\vec{\Delta s} = [\Delta s_x, \Delta s_y, 0]^T$ of $\vec{s}$ along S results in a change of $\vec{n}$, the outward normal to S, of

$$\vec{\Delta n} = \vec{n}(\vec{s} + \vec{\Delta s}) - \vec{n}(\vec{s}) = \begin{bmatrix} K \begin{bmatrix} \Delta s_x \\ \Delta s_y \end{bmatrix} \\ 0 \end{bmatrix} \quad (1)$$

The *principle curvatures* are the eigenvalues of the curvature matrix. The principle curvatures are always real because the curvature matrix is symmetric.

Let $\dot{\vec{s}}_1 = [\dot{s}_{1x}, \dot{s}_{1y}, 0]^T$ and $\dot{\vec{s}}_2 = [\dot{s}_{2x}, \dot{s}_{2y}, 0]^T$ be the rate at which the point of contact moves across the surfaces of obj_1 and obj_2 respectively. Let $\vec{v} = [v_x, v_y, v_z]^T$ and $\vec{\omega} = [\omega_x, \omega_y, \omega_z]^T$ be the translational and rotational velocities of $obj1$ relative to $obj2$, where the reference frame for $obj1$ is the local reference frame. Let K_1 and K_2 be the curvatures of $obj1$ and $obj2$ at the point of contact relative to the x and y axes of the local reference frame. Call the quantity $K_r = K_1 + K_2$ the *relative curvature* at the point of contact. The *contact constraint* $v_z = 0$ is a necessary and sufficient condition that the objects remain in contact. We can then derive the *contact equations*

$$\dot{\vec{s}}_1 = K_r^{-1}\left(\begin{bmatrix} -\omega_y \\ \omega_x \end{bmatrix} - K_2 \begin{bmatrix} v_x \\ v_y \end{bmatrix}\right), \quad (2)$$

$$\dot{\vec{s}}_2 = K_r^{-1}\left(\begin{bmatrix} -\omega_y \\ \omega_x \end{bmatrix} + K_1 \begin{bmatrix} v_x \\ v_y \end{bmatrix}\right) \quad (3)$$

For two different derivations of these equations, see [11] and [2].

4 A model of grasp dynamics

We now build a model of how the system (defined as the positions and velocities of the points of contact on the object surface) evolves in response to small disturbances from equilibrium.

4.1 Assumptions

The model utilizes the following underlying assumptions:

(1) There are exactly two points of contact (actually areas of contact due to the viscoelasticity of the fingertip), one for each of two fingers grasping the object.

(2) The fingertips maintain their grasp of the object. Hence, because frictional forces prevent slippage, the object can roll but not slip at the points of contact. (In the notation of Section 3, $v_x = v_y = \omega_z = 0$.)

(3) The evolution of the points of contact follow Equations 2-3 despite the fact that we are allowing the fingertips to be "soft", i.e. viscoelastic. The experimental results reported in [10, 12] indicate that this is reasonable when Assumption 2 holds.

(4) Disturbances from equilibrium are small enough that all first-order approximations are valid.

(5) The force and magnitude exerted by a fingertip is of constant magnitude and direction relative to its local reference frame. Therefore, as the fingertip rotates relative to the object, the direction of the applied force rotates likewise. Note that tactile sensing can tell us the position of the point of contact; hence, we could make the applied force a function of this tactile feedback in order to stabilize the system. However, for the purposes of this paper, we are interested in the natural stability of the system, i.e. the stability in the absence of any feedback response.

4.2 The Generated Torques

As the points of contact move across the object surface, there are three different types of torque generated: attractive, repulsive and dissipative. We describe the origin of each of these and derive its value.

Attractive torque: An attractive torque arises from the change in position at which a finger applies force. If the point of contact moves a small amount

$\vec{\Delta s} = [\Delta s_x, \Delta s_y]^T$ along the object's surface, then the vector from the object's center of mass to the point of contact changes by $\vec{\Delta r} = [\Delta s_x, \Delta s_y, 0]^T$ (measured relative to the object's contact frame). This produces an additional torque around the center of mass of

$$\tau_a = \vec{\Delta r} \times \vec{F} = \begin{bmatrix} \Delta s_x \\ \Delta s_y \\ 0 \end{bmatrix} \times \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} \quad (4)$$

where $\vec{F}$ is the force exerted by the finger on the object. Using Equation 2, we can rewrite this as

$$\tau_a = \begin{bmatrix} K_r^{-1} \begin{bmatrix} -\Delta\theta_y \\ \Delta\theta_x \end{bmatrix} \\ 0 \end{bmatrix} \times \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} \quad (5)$$

$$= \begin{bmatrix} F_z(J^T K_r^{-1} J) \begin{bmatrix} \Delta\theta_x \\ \Delta\theta_y \end{bmatrix} \\ \tau_{az} \end{bmatrix} \quad (6)$$

where K_r is the relative curvature at the point of contact, $\vec{\Delta\theta} = [\Delta\theta_x, \Delta\theta_y, 0]^T$ is the angular displacement (of the object relative to the finger) from equilibrium, $J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, and τ_{az} is a quantity about whose value we do not care because we have assumed that friction will compensate for this torque.

Note that (1) K_r always has positive, real eigenvalues due to physical constraints and hence so does K_r^{-1}, (2) $J^T K_r^{-1} J$ has the same eigenvalues as K_r^{-1}, and (3) F_z is always negative due to physical constraints. So, $F_z(J^T K_r^{-1} J)$ has negative, real eigenvalues, and it therefore always acts to oppose displacement from equilibrium in each of its principle directions. Hence, the torque is attractive.

Repulsive torque: The repulsive torque arises from a change in the direction of the applied force. With the assumption that the force is always applied in the same direction relative to the fingertip, the change in the direction of the applied force is $-\vec{\Delta\theta}$; therefore, the change in the force is $\Delta\vec{F} = -\vec{\Delta\theta} \times \vec{F}$. This produces an additional torque of

$$\tau_r = \vec{r} \times (-\vec{\Delta\theta} \times \vec{F}) = \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix} \times (- \begin{bmatrix} \Delta\theta_x \\ \Delta\theta_y \\ 0 \end{bmatrix} \times \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}) \quad (7)$$

$$= \begin{bmatrix} \begin{bmatrix} -r_z F_z - r_y F_y & r_y F_x \\ r_x F_y & -r_z F_z - r_x F_x \end{bmatrix} \begin{bmatrix} \Delta\theta_x \\ \Delta\theta_y \end{bmatrix} \\ \tau_{rz} \end{bmatrix} \quad (8)$$

where τ_{rz} is a quantity about which we do not care.

Note that the eigenvalues of the matrix $\begin{bmatrix} -r_z F_z - r_y F_y & r_y F_x \\ r_x F_y & -r_z F_z - r_x F_x \end{bmatrix}$ are $-r_z F_z$ and $-(r_x F_x + r_y F_y + r_z F_z) = -\vec{r} \cdot \vec{F}$. For most grasps, these eigenvalues are positive, and hence the torque is repulsive. However, there do exist situations for which one or both of these eigenvalues are negative, and hence the torque is partially or fully attractive thus belying its name.

Dissipative torque: The dissipative torque arises from the viscoelasticity of the fingers. This viscoelasticity produces a resistance to rolling, i.e. a torque around the point of contact which opposes the relative rotation of the two objects. Cutkosky and Wright [3] introduced rolling resistance into the robotics literature, acknowledging that it had previously been discussed at length in the literature on wheels and tires (see, e.g. [4]). We model the resulting torque on the object as $\tau_d = -\kappa_f F_n [\omega_x, \omega_y, 0]^T = \kappa_f F_z [\Delta\dot{\theta}_x, \Delta\dot{\theta}_y, 0]^T$, where $F_n = -F_z$ is the normal force and κ_f is a constant called the *coefficient of rolling resistance* [4].

4.3 The Equations of Motion

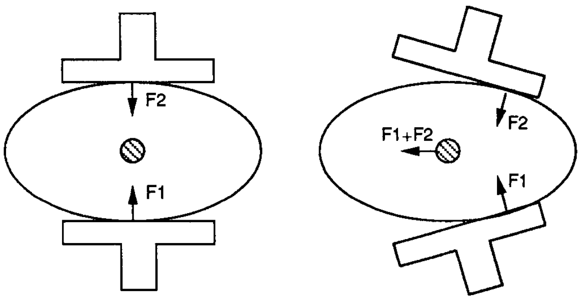
Let C_1 be a local reference frame for the grasped for the grasped object at one (equilibrium) point of contact and C_2 be a local reference frame at the other (equilibrium) point of contact. Let $(\vec{p}_0, R_0)$ be the rigid-body transformation which takes C_1 to C_2. Let $\tau_1 = \tau_{a1} + \tau_{r1} + \tau_{d1}$ and $\tau_2 = \tau_{a2} + \tau_{r2} + \tau_{d2}$ be the generated torques at the two points of contact. The local effect of these torques (i.e. the effect at the point of contact at which they are applied) is to generate angular accelerations $\ddot{\vec{\Delta\theta}}_{1L} = [\Delta\ddot{\theta}_{x1L}, \Delta\ddot{\theta}_{y1L}]^T$ and $\ddot{\vec{\Delta\theta}}_{2L} = [\Delta\ddot{\theta}_{x2L}, \Delta\ddot{\theta}_{y2L}]^T$

$$\ddot{\vec{\Delta\theta}}_{1L} = I_{23} M^{-1} \mathbf{diag}(1,1,0) \tau_1 = M_{22} I_{23} \tau_1 \quad (9)$$

$$\ddot{\vec{\Delta\theta}}_{2L} = I_{23}(R_0^T M^{-1} R_0) \mathbf{diag}(1,1,0) \tau_2 = \tilde{M}_{22} I_{23} \tau_2 \quad (10)$$

where $I_{23} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $M_{22} = I_{23} M^{-1} I_{23}^T$, $\tilde{M}_{22} = I_{23} R_0^T M^{-1} R_0 I_{23}^T$, and M is the inertia matrix of the object relative to a coordinate frame that is oriented the same as C_1 and whose origin is the center of mass of the object. (Recall from above that the z components of the torques are compensated for by friction.) The torques also cause angular acceleration at the other points of contact. The remote angular accelerations are

$$\ddot{\vec{\Delta\theta}}_{1R} = R_{22} \ddot{\vec{\Delta\theta}}_{2L}, \quad \ddot{\vec{\Delta\theta}}_{2R} = R_{22}^T \ddot{\vec{\Delta\theta}}_{1L} \quad (11)$$



(a) equilibrium => F1+F2 = 0 (b) displaced => F1+F2 acts along axis through center of mass

Figure 4: An eigenvector for eigenvalue 0.

where $R_{22} = I_{23} R_0 I_{23}^T$. Then, the total angular accelerations are

$$\ddot{\Delta\theta}_1 = \ddot{\Delta\theta}_{1L} + \ddot{\Delta\theta}_{1R}, \quad \ddot{\Delta\theta}_2 = \ddot{\Delta\theta}_{2L} + \ddot{\Delta\theta}_{2R}, \quad (12)$$

Substituting the values of the torques derived in Section 4.2 and writing the equations in matrix form, we get

$$\frac{d}{dt}\vec{g} = A\vec{g} \quad (13)$$

where $\vec{g} = [\Delta\theta_{x1}, \Delta\theta_{y1}, \Delta\theta_{x2}, \Delta\theta_{y2}, \dot{\Delta\theta}_{x1}, \dot{\Delta\theta}_{y1}, \dot{\Delta\theta}_{x2}, \dot{\Delta\theta}_{y2}]^T$ and where A is the 8x8 matrix

$$A = \begin{bmatrix} 0 & 0 & I_{22} & 0 \\ 0 & 0 & 0 & I_{22} \\ A_1 & R_{22}A_2 & A_3 & R_{22}A_4 \\ R_{22}^T A_1 & A_2 & R_{22}^T A_3 & A_4 \end{bmatrix} \quad (14)$$

where I_{22} is the 2x2 identity matrix and

$$A_1 = M_{22}(F_{z1} J^T K_{r1}^{-1} J - r_{z1} F_{z1} I_{22} + [r_{y1}, -r_{x1}]^T [-F_{y1}, F_{x1}]) \quad (15)$$

$$A_2 = \tilde{M}_{22}(F_{z2} J^T K_{r2}^{-1} J - r_{z2} F_{z2} I_{22} + [r_{y2}, -r_{x2}]^T [-F_{y2}, F_{x2}]) \quad (16)$$

$$A_3 = \kappa_f F_{z1} M_{22} \qquad A_4 = \kappa_f F_{z2} \tilde{M}_{22} \quad (17)$$

The system described by Equation 13 is stable if and only if the eigenvalues of the matrix A all lie in the left half-plane [9]. Hence, we have derived the condition for contact grasp stability; a grasp is contact stable if and only if the eigenvalues of A lie in the left half-plane.

4.4 Examples

We start by deriving the eigenvalues of A for the class of grasps satisfying the following conditions:

- $\vec{F}_1 = \vec{F}_2 = [0, 0, -F_n]^T$ (i.e. there is no tangential force at either point of contact)
- $\vec{r}_1 = \vec{r}_2 = [0, 0, r]^T$ (i.e. the center of mass is the centroid of the two points of contacts)
- $\vec{p}_0 = 2\vec{r}_1$ and $R_0 = \begin{bmatrix} R_{22} & 0 \\ 0 & -1 \end{bmatrix}$ where $R_{22} = \begin{bmatrix} \cos\psi & \sin\psi \\ \sin\psi & -\cos\psi \end{bmatrix}$ for some ψ (i.e. the points of contact are diametrically opposed),
- $K_{r2} = R_{22} K_{r1} R_{22}$ (note that $R_{22}^{-1} = R_{22}^T = R_{22}$)
- there exists a choice of the local reference frame C_1 which simultaneously diagonalizes K_{r1} and M.

With this choice of C_1, choose C_2 such that $R_{22} = \text{diag}(1, -1)$. Then, $K_{r1} = K_{r2} = \text{diag}(k_a, k_b)$, where k_a and k_b are the *principle relative curvatures.* Furthermore, we can write $M = \text{diag}(m_1, m_2)$. Computing the components of A gives

$$A_1 = A_2 = \text{diag}(m_1 F_n (r - k_b^{-1}), m_2 F_n (r - k_a^{-1})) \quad (18)$$

$$R_{22}^T A_1 = R_{22} A_2 = \text{diag}(m_1 F_n (r - k_b^{-1}), -m_2 F_n (r - k_a^{-1})) \quad (19)$$

$$A_3 = A_4 = \text{diag}(-m_1 F_n \kappa_f, -m_2 F_n \kappa_f) \quad (20)$$

$$R_{22}^T A_3 = R_{22} A_4 = \text{diag}(-m_1 F_n \kappa_f, m_2 F_n \kappa_f) \quad (21)$$

The eigenvalues of A are

$$\lambda_1 = \lambda_2 = 0, \quad \lambda_3 = -m_1 F_n \kappa_f, \quad \lambda_4 = -m_2 F_n \kappa_f \quad (22)$$

$$\lambda_5 = \frac{m_1 F_n}{2}(-\kappa_f - \sqrt{\kappa_f^2 - 4(k_b^{-1} - r)}) \quad (23)$$

$$\lambda_6 = \frac{m_1 F_n}{2}(-\kappa_f - \sqrt{\kappa_f^2 + 4(k_b^{-1} - r)}) \quad (24)$$

$$\lambda_7 = \frac{m_2 F_n}{2}(-\kappa_f - \sqrt{\kappa_f^2 - 4(k_a^{-1} - r)}) \quad (25)$$

$$\lambda_8 = \frac{m_2 F_n}{2}(-\kappa_f - \sqrt{\kappa_f^2 + 4(k_a^{-1} - r)}) \quad (26)$$

Hence, the grasp is contact stable if and only if $k_a^{-1} \geq r$ and $k_b^{-1} \geq r$.

Note that the eigenvalues $\lambda_1 = 0$ and $\lambda_2 = 0$ define a two-dimensional subspace of eigenvectors spanned by $\vec{v}_1 = [1, 0, 1, 0, 0, 0, 0, 0]^T$ and $\vec{v}_2 = [0, 1, 0, -1, 0, 0, 0, 0]^T$. Vectors in this subspace correspond to states of the system where the two points of contact are stationary and displaced the same amount in the same absolute direction. As shown in Figure 4, in such a state there is no net torque. This borderline instability is not harmful because the velocity components are zero, and hence it does not cause displacements from equilibrium to increase in magnitude.

Example 5 Let the grasped object be an ellipsoid given by the equation $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$. Let the finger surfaces be flat. Let the points of contact be $(a, 0, 0)$ and $(-a, 0, 0)$. Then, the principle relative curvatures are $k_a = 1/b$ and $k_b = 1/c$, and the distance from the center of mass to each point of contact is $r = a$. Hence, the grasp is stable if $b > a$ and $c > a$. This explains the intuitive assessments of Examples 1 and 3.

Example 6 Let the grasped object be a cube with sides of length d. Let the fingertips be spherical with radius R. Let the points of contact be at the centers of two opposite faces. Then, the principle relative curvatures are $k_a = k_b = 1/R$, and the distance is $r = d/2$. Hence, the grasp is stable if $R > d/2$. This gives some insight into why humans often use the less curved part of their fingertips for grasping bigger objects and the more curved part for grasping smaller objects.

5 Conclusion

We have distinguished between two types of grasp stability, spatial grasp stability and contact grasp stability, each with a different concept of the state of a grasp. The previous work on quantifying grasp stability has focused primarily on spatial stability. Using examples, we have shown that there are differences in the stability of grasps which cannot be captured using any measure of spatial stability and hence that any full understanding of grasp stability must also involve contact stability. Therefore, we have derived a quantitative measure of contact grasp stability. To do this, we have formulated a model of the dynamics of two-fingered grasps, where the state of a grasp is the position (and velocity) of the points of contact. This model is built on concepts and results from the study of contact kinematics. The resulting equations of motion are linear; therefore, the system is stable if and only if the eigenvalues of the matrix which determines its evolution lie in the left half-plane. While the general form of this matrix is complex and does not admit a closed-form for its eigenvalues, we have calculated the eigenvalues for some examples. In these cases, the derived stability corresponds well with intuitive notions of grasp stability.

References

[1] R.W. Brockett. Robotic hands with rheological surfaces, *Proc. IEEE Conf. Robotics and Automation*, pp. 942–946, 1985.

[2] C. Cai and B. Roth. On the spatial motion of rigid bodies with point contact, *Proc. IEEE Conf. Robotics and Automation*, pp. 686–695, 1987.

[3] M.R. Cutkosky and P.K. Wright. Friction, stability and the design of robotic fingers, *Int. J. Robotics Research,* vol. 5, no. 4, pp. 20–37, 1986.

[4] J. Halling. *Principles of Tribology,* London, Macmillan, 1975.

[5] H. Hanafusa and H. Asada. Stable prehension by a robot hand with elastic fingers, *Proc. 7th Int. Symposium on Industrial Robots*, pp. 361–368, 1977

[6] P. Hsu, Z. Li and S. Sastry. On grasping and coodinated manipulation by a multifingered robot hand, *Proc. IEEE Conf. Robotics and Automation*, pp. 384–389, 1988.

[7] J. Kerr and B. Roth. Analysis of multifingered hands, *Int. J. Robotics Research*, vol. 4, no. 4, pp. 3–17, 1986.

[8] Z. Li and S. Sastry. Task oriented optimal grasping by multifingered robot hand, *Proc. IEEE Conf. Robotics and Automation*, pp. 389–394, 1987.

[9] J.C. Maxwell. On governors, *Proc. of the Royal Society*, No. 100, 1868.

[10] D.J. Montana. *Tactile Sensing and the Kinematics of Contact*, Ph.D. Thesis, Harvard University, Division of Applied Sciences, August, 1986.

[11] D.J. Montana. The kinematics of contact and grasp, *Int. J. Robotics Research*, vol. 7, no. 3, pp. 17–32, 1988.

[12] D.J. Montana. The kinematics of contact with compliance, *Proc. IEEE Conf. Robotics and Automation*, pp. 770–774, 1989.

[13] V. Nguyen. Constructing stable grasps, *Int. J. Robotics Research*, vol. 8, no. 1, pp. 26–37, 1989.

[14] J.S. Pettinato and H.E. Stephanou. Manipulability and stability of a tentacle based robot manipulator, *Proc. IEEE Conf. Robotics and Automation*, pp. 458–463, May, 1989.

[15] J.K. Salisbury. *Kinematic and Force Analysis of Articulated Hands*, Ph.D. thesis, Stanford University, Dept. of Mechanical Engineering, May, 1982.